Statistical Result And Theoretical Interpretation of Σ-D Relation for Shell-Type Galactic Supernova Remnants

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ABSTRACT

In this paper, after collecting 57 shell-type galactic supernova remnants data, we first make the statistical result of Σ-D relation on these data by applying the linear regression method. Our statistical result shows that the best fit line slope of Σ-D relation is $\beta = -2.56$, which is slightly flatter than those proposed by some other authors before. In addition, comparing with the statistical results of Σ-D relation by some other authors, we can find that a transition point is usually introduced in their statistical results. In order to make a theoretical interpretation for this transition point in the statistical results of Σ-D relation, we also analytically investigate the Σ-D relations of shell-type supernova remnants at 1 GHz both at the adiabatic phase and radiative phase, which is simply followed the work Duric & Seaquist (1986). Our analytical results show that indeed there can be a transition point at 1 GHz between these two analytical Σ-D relations. Moreover, the analytical transition point is 30 pc, which can be consistent with the statistical results at 1 GHz made by some other authors before, i.e. 32 pc at 1 GHz made by Allakhverdiyev et al. (1983).

Subject headings: methods: — statistics — analysis— (ISM:) supernova remnants

1. Introduction

The relation between radio surface brightness (Σ) and diameter (D) of supernova remnants (SNRs) is usually used to determine the distance of a SNR (Poveda & Woltjer 1968; Clark & Caswell 1976; Lozinskaya 1981; Huang & Thaddeus 1985; Duric & Seaquist 1986; Guseinov et al. 2003), thus it has been widely discussed in many works via statistical or analytical approaches (e.g., Poveda & Woltjer 1968; Clark & Caswell 1976; Mills et al. 1984; Huang & Thaddeus 1985; Arbutina et al. 2004, Pavlovic et al. 2014, etc.). Among the statistical results of Σ-D relation, one straight line was often obtained by authors (e.g., Poveda & Woltjer 1968; Huang & Thaddeus 1985; Arbutina et al. 2004; Pavlovi et al. 2014). In our paper, after collecting 57 shell-type galactic supernova remnants data where some data have been updated according to Green (2004, 2009 & 2014) and other new references, we also make the statistical result of Σ-D relation on these data by simply using the linear regression method (Pavlovic et al. 2013). Here, the dominant selection effects are those that are applicable at radio wavelengths and same as Green (2004, 2009 & 2014). Our statistical result shows that the best fit line slope of Σ-D relation with a straight line is $\beta = -2.56$, which is slightly flatter than those proposed by some other authors before.

Note that, however, a broken fit line or a transition point is also usually seen in the statistical results. For example, Clark & Caswell (1976), Allakhverdiyev et al. (1983), and Allakhverdiyev et al. (1985) have gotten a broken fit line in their statistical works. At 408 MHz, Clark & Caswell (1976) had a broken line with slopes of $\beta = -2.7/-10$ ($\Sigma \propto D^{\beta}$) at $D \leq 32$ pc/$D \geq$

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32 pc, while Allakhverdiyev et al. (1983) got 30 pc at 408 MHz and 32 pc at 1 GHz for 15 shell-type remnants. But yet, the research is few for the analytical interpretation on the broken line or transition point in the Σ-D relation. Duric & Seaquist (1986) once made that

$$\Sigma(D) = 4 \times 10^{-14} D^{-5}, D \ll 1pc \tag{1}$$

$$\Sigma(D) = 4 \times 10^{-15} D^{-3.5}, D \gg 1pc \tag{2}$$

which analytically predicted that there was a transition point for the Σ-D relation of supernova remnants. But it should be truly said that the analytical interpretation on the broken line or transition point is still lost, i.e., the exact analytical value of transition point is still absent. In order to fill this gap, we should further analytically investigate the Σ-D relation of supernova remnants.

On the other hand, the galactic SNRs are usually classified into three types: Shell-type, Plerion-type and Composite-type. In our paper, for the simplicity, we just focus on investigating the shell-type galactic supernova remnants. For the shell-type galactic supernova remnants, they usually have four evolution stages: the free expansion phase, the adiabatic or Sedov phase, the radiative or snowplough phase and the dissipation phase. In addition, nearly all of the detected shell-type SNRs are at the adiabatic phase or the radiative phase, because almost none is observed in the 1st and 4th phases due to the fact that the shell-type SNRs at these two phases are usually practically undetectable. Therefore, a direct conjecture is that the theoretical interpretation for this transition point in the statistical results of Σ-D relation may come from the phase transition between these two detectable shell-type SNRs. After the analytically investigation on the Σ-D relations both at the adiabatic phase and radiative phase of shell-type supernova remnants at 1 GHz, which is simply followed the work Duric & Seaquist (1986), our results show that indeed there can be a transition point between these two analytical Σ-D relations in 30 pc at 1 GHz. Moreover, this exact analytical value of transition point can be consistent with the statistical results made by some other authors before, i.e. 32 pc at 1 GHz made by Allakhverdiyev et al. (1983).

The rest of our paper is organized as follow. In section 2, after collecting 57 shell-type galactic supernova remnants data at 1 GHz, we have made the statistical result of Σ-D relation on these data, where the best fit line slope with a straight line is $\beta = -2.56$. In section 3, after a brief review of the work Duric & Seaquist (1986), we simply follow their work and further analytically investigate the Σ-D relation at the radiative phase of shell-type galactic supernova remnants at 1 GHz. Moreover, after several discussions, the transition point can analytically exist in 30 pc at 1 GHz between these two stages, which can be consistent with the statistical results. Finally, a brief conclusion and discussion are given in section 4.

2. Statistical result of Σ-D relation

We have collected 57 shell-type supernova remnants data in Galaxy at 1 GHz and listed them in table 1, where some data have also been updated according to the new references. In order to make statistical result of Σ-D relation on these data, we first obtain the surface brightness Σ_{1GHz} from this table, and the 1GHz surface brightness Σ_{1GHz} is given by (Clark & Caswell 1976)

$$\Sigma_{1GHz} = 1.505 \frac{S_{1GHz}}{\theta^2} \times 10^{-19} \ (Wm^{-2}Hz^{-1}sr^{-1}), \tag{3}$$

where S_{1GHz} is the 1GHz flux density in jansky ($1Jy \equiv 10^{-26} Wm^{-2}Hz^{-1}$), and θ is the angular diameter in minutes of arc. Then, we simply apply the linear regression method, i.e., if the aim function is $y(x) = a + bx$, the parameters a and b can be written as

$$b = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}, \tag{4}$$

$$a = \overline{y} - b\overline{x}. \tag{5}$$

Therefore, one can obtain

$$\ln \Sigma_{1GHz} = 8.97 - 2.56 \ln D, \tag{6}$$

which has been plotted in Fig. 1. Note that, for the convenience of plotting the figure, we have used the ln function, and the units of Σ_{1GHz} and D are $\times 10^{-22} (Wm^{-2}Hz^{-1}sr^{-1})$ and pc, respectively. In addition, since our investigations in the following mainly focus on the surface brightness at 1GHz, we will often ignore the bottom index in Σ_{1GHz} for our results, while sometimes D is also expressed its unit in pc.

From the Eq.(6), the final statistical result with a straight line on these data is

$$\Sigma(D) = 7.85 \times 10^{-19} D_{pc}^{-2.56} \ (Wm^{-2}Hz^{-1}sr^{-1}). \quad (7)$$

Comparing with other statistic results, i.e., Case & Bhattacharya (1998) got a straight line

$$\Sigma(D) = 5.43 \times 10^{-17} D_{pc}^{-2.64} \ (Wm^{-2}Hz^{-1}sr^{-1}). \quad (8)$$

and Xu et al. (2005) derived a straight flatter line

$$\Sigma(D) = 1.21 \times 10^{-18} D_{pc}^{-1.60} \ (Wm^{-2}Hz^{-1}sr^{-1}). \quad (9)$$

we can find that all these corresponding best fit values, i.e. $\beta = -2.56, -2.64, -1.6$, are flatter than those derived by some authors at early time, which can reach as high as $\beta = -5.2$ at 1 GHz (Pavlovi et al. 2014), $\beta = -6$ (Allakhverdiyev et al. 1985) and $\beta = -10$ (Clark & Caswell 1976).

3. Theoretical interpretation of Σ-D Relation

It should be pointed out that, the Σ-D relation with a broken line or transition point is also usually seen in the statistical results. For example, Clark & Caswell (1976) got a transition point about 32 pc in the diameter for 29 galactic SNRs at 408 MHz, and 32 pc at 5000 MHz. Allakhverdiyev et al. (1983) got 30 pc at 408 MHz for 15 shell-type remnants, and 32 pc at 1 GHz. For a larger number of samples of 146 all-sort galactic objects including plerion, shell and composite-type remnants, Allakhverdiyev et al. (1985) obtained 40 pc at 1 GHz. Therefore, making the statistical result of Σ-D relation for our collected

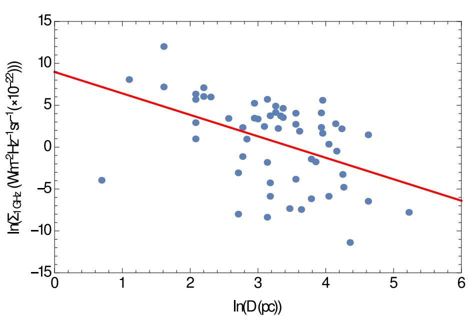

Fig. 1.— For 57 Shell-type Galactic SNRs, the surface brightness (Σ) decreases with their linear diameter (D) by a slope of about −2.56.

57 shell-type supernova remnants data with a broken line will be interesting. However, since there have been many statistical works with broken line referred to in the above, thus giving a theoretical interpretation of Σ-D relation with a broken line or transition point will be more interesting and important. In the following, we will give a simple theoretical interpretation for this transition point. For the simplicity, we just focus on the shell-type galactic SNRs. Note that, nearly all of the detected shell-type SNRs are at the adiabatic phase or the radiative phase. In order to give a theoretical interpretation of Σ-D relation with a transition point, we need further analytically investigation on the Σ-D relations both at these two phases of shell-type SNRs, which we just simply follow the work Duric & Seaquist (1986). For the convenience of comparison and making the whole paper more readable, here we also do a warmup to make a brief review on the work Duric & Seaquist (1986) in this section, where they just analytically investigate the adiabatic phase.

3.1. A brief review : work Duric & Seaquist (1986)

Taking the linear diameter (D) of remnant in pc, time (t) in s, SNR initial explosion energy (E_0) in the unit of ergs, and ISM electron density (n_0) in cm^{-3}, from the standard Sedov solution, one has the following equation (Bignami & Caraveo 1988, Zaninetti 2000, Völk et al. 2002, Ptuskin & Zirakashvili 2003)

$$D(t) = A_0 t^{2/5}, \quad (10)$$

where the coefficient

$$A_0 = 6.3 \times 10^{-4} \left(\frac{E_0}{n_0} \right)^{1/5}. \quad (11)$$

The shock wave velocity should be

$$v(t) = \frac{1}{2}\frac{d}{dt}D(t) = \frac{1}{5}A_0 t^{-3/5}. \quad (12)$$

At the adiabatic phase, the thickness of remnant is proportional to D (Milne 1970), and the shell volume which contains all the radio-emitting particles is

$$V(D) = C_0 D^3, \quad (13)$$

here $C_0 = \frac{\pi}{6}(1 - (\frac{D_i}{D_0})^3) \simeq 0.37$ is the volume coefficient. Notice that condition $D_i/D_o \sim 2/3$

has been assumed and D_i and D_o are the inner and outer diameter of the remnant shell respectively. Combining (10) and (13), one obtains the volume of the shell with respect to t

$$V(t) = C_0 A_0^3 t^{6/5}. \tag{14}$$

As the shock waves of remnant travel, the ambient magnetic field B at the adiabatic phase will decrease with D according to (Duric & Seaquist 1986)

$$B(D) = B_0 \left(\frac{D_0}{D} \right)^2. \tag{15}$$

Substituting (10) to it, we have

$$B(t) = B_0 D_0^2 A_0^{-2} t^{-4/5}. \tag{16}$$

Ginzburg & Synovatskii (1965) and bell (1978) show that the radio emissivity $\epsilon(B, \upsilon)$ of a shocked gas which are affected by a magnetic field to produce the synchrotron emission can be expressed as (Arbutina B. et al. 2012)

$$\begin{aligned} \epsilon(\nu) = & \, 2.94 \times 10^{-34} (1.435 \times 10^5)^{0.75-\alpha} \xi(2\alpha + 1) \\ & \times \left(\frac{n_0}{cm^{-3}}\right) \left(\frac{\alpha}{0.75}\right) \left(\frac{\upsilon}{10^4 \ km \ s^{-1}}\right)^{4\alpha} \left(\frac{B}{10^{-4} G}\right)^{\alpha+1} \\ & \times \left(1 + \left(\frac{\upsilon}{7000 \ Km \ s^{-1}}\right)^{-2}\right)^{\alpha} \left(\frac{\nu}{GHz}\right)^{-\alpha}, \end{aligned} \tag{17}$$

where the unit of $\epsilon(\nu)$ is $WHz^{-1}m^{-3}$, $\xi(\mu) = 11.7a(\mu)$, and $a(\mu)$ is the function tabulated by Ginzburg & Synovatskii (1965). The velocities of shock waves in the second and third phase of SNRs are typically far less than 7000 $Km \ s^{-1}$. Thus, (17) can be further simplified

$$\begin{aligned} \epsilon(\nu) = & \, 2.94 \times 10^{-34} \times (1.435 \times 10^5)^{0.75-\alpha} \xi(2\alpha + 1) \\ & \left(\frac{\alpha}{0.75}\right)(0.7)^{4\alpha}\left(\frac{n}{cm^{-3}}\right)\left(\frac{v_s}{7000km/s}\right)^{2\alpha}\left(\frac{B}{10^{-4}G}\right)^{\alpha+1}\left(\frac{\nu}{GHz}\right)^{-\alpha} \\ & (WHz^{-1}m^{-3}). \end{aligned} \tag{18}$$

Taking account of (10), (16) and the average value of the remnants spectral index $\alpha = 0.5$, we can get

$$\begin{aligned} \epsilon(D) = & \, 2.25 \times 10^{-34} \left(\frac{D_0}{D} \right)^3 \left(\frac{B_0}{10^{-4} G} \right)^{3/2} \\ & \times \left(\frac{1}{5} A_0^{5/2} D^{-3/} / 7000 Km \ s^{-1} \right). \end{aligned} \tag{19}$$

which is at 1 GHz. If the shell volume is considered to be encompassed by the radiating electrons, the surface brightness of remnant can be written as (Duric & Seaquist 1986)

$$\Sigma(t) = \frac{\epsilon(t) V(t)}{\pi^2 D^2(t)}. \tag{20}$$

Inserting (10) (11) (14) and (19) into it, we obtain

$$\begin{aligned} \Sigma(D) = & \, 2.25 \times 10^{-34} \frac{C_0 D_0^3}{\pi^2 D^2} \left(\frac{B_0}{10^{-4} G} \right)^{3/2} \\ & \times \left(\frac{1}{5} A_0^{5/2} D^{-3/2} / 7000 Km \ s^{-1} \right). \end{aligned} \tag{21}$$

Finally, one can get

$$\Sigma(D) = m_a D_{pc}^{-3.5} \ (Wm^{-2}Hz^{-1}sr^{-1}), \tag{22}$$

where

$$\begin{aligned} m_a & = 2.25 \times 10^{-34} \frac{C_0 D_0^3}{\pi^2} \left(\frac{B_0}{10^{-4} G} \right)^{3/2} \\ & \times \left(\frac{1}{5} A_0^{5/2} / 7000 Km \ s^{-1} \right) \\ & = 5.76 \times 10^{-17}, \end{aligned} \tag{23}$$

here some typical values of physical parameters of SNRs are taken: ISM density $n_0 = 0.1$ cm^{-3}, SNR initial explosion energy $E_0 = 10^{51}$ erg, the diameter and ISM magnetic field of remnant at the beginning of Sedov phase $D_0 = 2$ pc and $B_0 = 10^{-4}$ G, etc. Therefore, the analytically derived line of Σ-D relation at the second phase of shell-type SNR can be

$$\Sigma(D) = 5.76 \times 10^{-17} D_{pc}^{-3.5} \ (Wm^{-2}Hz^{-1}sr^{-1}). \tag{24}$$

Note that, in order to keep consistent with the following analytical investigation on the radiative phase of shell-type, we have chosen different typical values. Thus the coefficient in (24) is different from that in work Duric & Seaquist 1986, but the power-law reminds the same slope.

3.2. Analytical Σ-D relation at the radiative phase

It should be pointed out that the above work Duric & Seaquist (1986) just analytically investigated the adiabatic phase of shell-type SNRs. In fact, we can also simply follow their work to analytically investigate the Σ-D relation at the radiative phase of shell-type SNRs. After setting the same choices of units as those in section (3.1), the equation for shell-type SNRs at the radiative stage is (Mckee et al. (1977))

$$D(t) = A_1 t^{2/7}, \tag{25}$$

where A_1 is a constant

$$A_1 = 0.03 \left(\frac{E_0}{n_0} \right)^{1/7}. \tag{26}$$

From which, we obtain the velocity of shock wave at the radiative phase

$$\upsilon(t) = \frac{1}{7}A_1 t^{-5/7}. \quad (27)$$

Same as the adiabatic phase, the volume of shell can be

$$V(D) = C_1 D^3. \quad (28)$$

If we roughly take $D_i/D_o \sim 3/4$, then the coefficient will be $C_1 = \frac{\pi}{6}(1-(\frac{D_i}{D_0})^3) \simeq 0.3$. Changing the variant D to t, one can rewrite the volume of shell as

$$V(t) = C_1 A_1^3 t^{6/7}. \quad (29)$$

Note the truth that the ambient magnetic field B of a remnant decreasing with the diameter D at the adiabatic phase is (15), while at the dissipation-phase it is $B(D) = B_1(D_1/D)^0$. Therefore, we can moderately suppose that the ambient magnetic field B at the radiative phase can be

$$B(D) = (\frac{D_1}{D})^\beta B_1, \quad (30)$$

where the parameter β ranges from 0 to 1. After substituting (25) to it, one gets

$$B(t) = \left(\frac{D_1}{A_1}\right)^\beta B_1 t^{-2/7}. \quad (31)$$

Therefore, following the same steps as the above section and still taking $n_0 = 0.1$ cm^{-3}, we can obtain the $\Sigma - D$ relation at the radiative phase at 1 GHz

$$\begin{aligned}\Sigma(D) &= 2.25\times10^{-37}\frac{C_1 D^3}{\pi^2 D^2}\left(\ \frac{B_1 D_1^\beta D^{-\beta}}{10^{-4}G}\ \right)^{3/2} \\ &\times \left(\ \tfrac{1}{7}A_1^{7/2}D^{-5/2}/7000Km\ s^{-1}\ \right). \quad (32)\end{aligned}$$

Now we have the form

$$\Sigma(D) = m_r D^{-\frac{3}{2}(1+\beta)}\ (Wm^{-2}Hz^{-1}sr^{-1}), \quad (33)$$

where

$$\begin{aligned}m_r &= 2.25\times10^{-37}\frac{C_1 D_1^{\frac{3}{2}\beta}}{\pi^2}\left(\ \frac{B_1}{10^{-4}G}\ \right)^{3/2} \\ &\times \left(\ \tfrac{1}{7}A_1^{7/2}/7000Km\ s^{-1}\ \right). \quad (34)\end{aligned}$$

3.3. Analytical transition point of Σ-D relations between the adiabatic phase and radiative phase

In order to discuss the transition point of Σ-D relations between the adiabatic phase and radiative phase, we set all the parameters at the radiative phase as their typical values, $B_1 = 10^{-5}$G, $D_1 = 20$ pc, $\beta = 1$. Then, the Σ-D relation at the radiative phase is

$$\Sigma(D) = 5.38\times10^{-17}D_{pc}^{-3}\ (Wm^{-2}Hz^{-1}sr^{-1}). \quad (35)$$

However, here it arises an interesting question that the initial typical value of diameter $D_1 = 20$ pc in (30) at the radiative phase is not equal to the transition value of Σ-D relations between the adiabatic phase and radiative phase. Note that, the exact transition value explicitly depends on the choice of typical values, i.e. B_1, D_1 and so on. In order to clearly show this question and for the simplicity, here we investigate the dependence of exact transition value just on the initial value of diameter D_1, i.e. varying D_1 and keeping other parameters as constants. In addition, for further convenient, we sign D_t as the transition diameter of Σ-D relations between the adiabatic phase and radiative phase, which satisfies

$$m_a D_t^{-3.5} = m_r D_t^{-3}. \quad (36)$$

After investigating (33) and (34), we can find that the coefficient m_r is sensitive to the initial value of diameter D_1 at the radiative phase. In fact, in the case of (35), the D_t is 108.2 pc, which is larger than 20 pc. If we choose the $D_1 = 36$ pc, we can obtain $D_t = 18.6$ pc. However, the difference is that $D_t < D_1$ in this case, while $D_t = 108.2\,pc > D_1 = 20\,pc$ in the previous result. These two results can implicate that the radio surface brightness of SNR may have a bound when the SNR transits from adiabatic phase to radiative phase. Note that, there is another case such that $D_t = D_1$ if we choose an appropriate value of D_1. After expressing D_t as the function of D_1, we can obtain

$$D_t = 8.37\times10^5 D_1^{-3}, \quad (37)$$

where D_t and D_1 are in the unit of pc. Thus if $D_t = D_1$, one can easily derive the appropriate D_1

$$D_t = D_1 = 30\ pc. \quad (38)$$

After compared with the statistics results, i.e., Allakhverdiyev et al. (1983) obtained 32 pc at 1 GHz, it seems that the most possible case is that there is no bound but a transition point at $D_t = 30\,pc$ for the radio surface brightness of SNR when the SNR transits from adiabatic phase to radiative phase.

4. Conclusion and Discussion

In this paper, we have collected 57 shell-type SNRs data in Galaxy among the references, and made the statistical result of Σ-D relation on these data by simply using the linear regression method, where the best fit line slope with a straight line is $\beta = -2.56$. For further giving a theoretical interpretation of broken line of Σ-D relation by some other statistical results, we also carefully analytically investigate the Σ-D relations both at the adiabatic phase and radiative phase. We find that there indeed can be an analytical transition point between these two phases. Moreover, this exact analytical value of transition point can be consistent with some statistical results. Therefore, two significant consequence can also be concluded. The first is that this transition point can be produced by the phase transition of Shell-type SNRs. The second is that the empirical relation with a transition point for Σ-D relation via statistics may be more applicable.

Note that, some progresses of the investigations on the statistical or theoretical Σ-D relations have also been made in the last several decades. For example, Pavlovic et al.(2013) have referred to that there have been some new regression methods to make the statistical results, i.e. double regression and orthogonal regression, while Berezhko & Volk (2004) have proposed new analysis on the theoretical interpretation of Σ-D relation. In our paper, since we just first collect and update the SNRs data according to the new references, and then mainly focus on giving a theoretical interpretation of Σ-D relation with a transition point, thus here we all just use the simple analyses, i.e. simply using the linear regression method to make the statistics and just generalizing the work Duric & Seaquist (1986) to the radiative phase. It should be pointed out that our simple analyses seems to be enough viewed from our results, i.e., indeed we have obtained the transition point from the theoretical investigation and furthermore this transition point can be consistent with the statistical result. Moreover, we can conclude that the empirical relation with a transition point for Σ-D relation via statistics maybe be more applicable. However, it will be still interesting in the future work to have further investigations on our statistical and theoretical results by taking the new analyses into account.

Some other discussions related to our results in this paper are also ordered. First, it should be pointed out that the transition point D_t is sensitive to those parameters such as the volume coefficient C_0, mean electron density n_0, SNR initial explosion energy E_0, magnetic field at the beginning of the evolving second-stage and third-stage B_0 and B_1, and the parameters D_0 and D_1. Meanwhile, the true physical process of the third stage of supernova remnant is complicated. Therefore, for the simplicity, we just consider the synchrotron radiation equation (18) is still valid in the third stage, and only take the effect of D_1 into account in the above to obtain the transition point. In fact, we can also take the effects of other parameters into account, i.e. the effect of the electron density n_0. Note that, throughout the paper we take the electron density $n_0 = 0.1$ cm^{-3} at both the adiabatic phase and radiative phase. One can see that the electron density n_0 denotes the density of electrons inside SNR shell which emits synchrotron radiation according to (17). These electrons can come from not only the interstellar media (ISM) with particle density typically equals to 0.1 cm^{-3}, but also the SNR progenitor. Thus the electron density will be truly larger than 0.1 cm^{-3}, and hence the value of D_t obtained will also be different. Therefore, the further effects of other parameters on the exact transition point will be an interesting open issue.

In addition, we would like to discuss the validity of the equation (18). Obviously, it is valid just when the velocity of shock wave is much less than 7000 km/s. According to (10) (12), the corresponding diameter at the second phase should be much larger than 10.6 pc when $E_0 = 10^{51} erg$ and $n_0 = 0.1$ cm^{-3}. And when $v_s = 7000/\sqrt{10}\ km/s$, the corresponding diameter is 22.9 pc (if SNR is still at the second phase) which is close to the transition point. Therefore, according to the above analysis, the equation (18) is valid at least in the

end of the second phase, which shows that the transition point $D_t = 30\ pc$ is reasonable. Of course, if the initial values of SNR such as E_0, n_0 are different, the discussion may also be changed, which can be further studied.

Finally, an interesting result may also be implicated from our analytical results of the Σ-D relations both at the adiabatic phase and radiative phase, which is that a new state may exist between the adiabatic phase and radiative phase for the shell-type SNRs. Since the time at the transition point $D_t = 30pc$ for the adiabatic phase and radiative phase from the equations (10) and (25) is $t_1 = 27.9$yr and $t_2 = 3387.8$yr respectively, which is obvious that $t_1 \leq t_2$. Note that, the typical lifetime of adiabatic phase and radiative phase are 10^4 yr and 10^9 yr, and t_2 is close to the typical lifetime of adiabatic phase 10^4 yr. Therefore, we conjecture that $D(t)$ maybe can remain constant, i.e, $D(t) = 30$ pc when t ranges from t_1 to t_2. In other words, there may be a stable state between the adiabatic phase and radiative phase. However, the situation is obviously weird that the SNR diameter can stay constant (=30 pc) for ages ranging from 27.9 yr to 3387.8 yr, therefore, the underlying physics on how the SNR expansion could stop from some time to restart later on and the experimental observation are needed to be further found out.

5. Acknowledgments

Y.P Hu thanks Profs.Hongsheng Zhang and Nan Liang for useful discussions and Dr. Ai-Yuan Yang for providing some updated data. This work is supported by the Fundamental Research Funds for the Central Universities under grant No. NS2015073, National Natural Science Foundation of China (NSFC) under grants No.11105004, 11575083, 11565017 and Shanghai Key Laboratory of Particle Physics and Cosmology under grant No. 11DZ2260700. In addition, this work is also supported by the Open Project Program of State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, China (No. Y5KF161CJ1). Jun Fang is supported by NSFC No. 11563009 and Yunnan Applied Basic Research Projects No. C0120150289.

REFERENCES

Allakhverdiyev A.O., Amnuel P.R., Guseinov O.H., Kasumov F.K. 1983, Ap&SS, 97, 287

Allakhverdiyev A.O., Guseinov O.H., Kasumov F.K., Yusifov I.M. 1985, Ap&SS, 121, 21

Arbutina B., Urosevic D., Stankovic M., Tesic Lj. 2004, MNRAS, 350, 346

Arbutina B., Urosevic D., Andjelic M.M., Pavlovic M.Z., Vukotic B., 2012, ApJ, 746, 79

Aschenbach B., Iyudin A.F., Schönfelder V. 1999, A&A, 350, 997 (AIS99)

Bell, A. R. 1978, MNRAS, 182, 443

erezhko, E. G., Völk, H. J. 2004, A&A, 427, 525

Berthiaume G.D., Burrows D.N., Garmire G.P., Nousek J.A. 1994, ApJ, 425, 132 (B94)

Bignami, G. F., P. A. Caraveo, and J. A. Paul. 1988, A&A, 202, L1-L4

Binette L., Dopita M.A., Dodorico S., Benvenuti P. 1982, A&A, 115, 315 (B82)

Carter L.M., Dickel J.R., Bomans D.J. 1997, PASJ, 109, 990 (CDB97)

Clark D.H., Caswell J.L. 1976, MNRAS, 174, 267

Case G.L., Bhattacharya D. 1998, ApJ, 504, 761

Case G.L., Bhattacharya D. 1999, ApJ, 521, 246 (CB99)

Caswell J.L., Haynes R.F., Milne D.K., Wellington K.J. 1982, MNRAS, 200, 1143 (C82)

Chen Y., Slane P.O. 2001, ApJ, 563, 202 (CS01)

Chen Y., Sun M., Wang Z.R., Yin Q.F. 1999, ApJ, 520, 737 (C99)

Dickel J.R., Strom R.G., Milne D.K. 2001, ApJ, 546, 447 (DSM01)

Dubner G.M., Moffett D.A., Goss W.M., Winkler P.F. 1993, AJ, 105, 2251 (D93)

Duncan A.R., Stewart R.T., Campbell-Wilson D., Haynes R.F., Aschenbach B., Jones K.L. 1997, MNRAS, 289, 97 (D97)

Duric N., Seaquist E.R. 1986, ApJ, 301, 308

Fesen R.A., Horford A.P. 1995, AJ, 110, 747 (FH95)

Fürst E., Reich W., Steube R. 1984, A&A, 133, 11 (FRS84)

Gaensler B.M., Johnston S. 1995, MNRAS, 277, 1243 (GJ95)

Galas C.M.F., Tuohy L.R., Garmire G.P. 1980, ApJ, 236, L13 (GTG80)

Ginzburg, V. L., Syrovatskii, S. I. 1965, Annual Review of Astronomy and Astrophysics 3 : 297

Gorham P.M., Ray P.S., Anderson S.B., Kulkarni S.R., Prince T.A. 1996, ApJ, 458, 257 (G96)

Green D.A. 2004, arXiv:astro-ph/0411083vl, 3 (G04a)

Green D.A. 2009, arXiv:0905.3699

Green D.A. 2014, arXiv:1409.0637v1 (G14)

Guseinov O.H., Ankay A., Sezer A., Tagieva S.O. 2003, A&AT, 22, 273 (G03)

Hatsukade I., Tsunemi H., Yamashita K., Koyama K., Asaoka Y., Asaoka I. 1990, PASJ, 42, 279 (H90)

Huang Y.-L., Thaddeus P. 1985, ApJ, 295, L13

Hughes V.A., Harten R.H., van den Bergh S. 1981, ApJ, 246, L127 (HHv81)

Kargaltsev O., Pavlov G.G., Sanwal D., Garmire G.P. 2002, ApJ, 580, 1060 (K02)

Kinugasa K., Torii K., Tsunemi H., Yamauchi S., Koyama K., Dotani T. 1998, PASJ, 50, 249 (K98a)

Knöedlseder J., Oberlack U., Diehl R., Chen W., Gehrels N. 1996, A&AS, 120, 339 (K96)

Koo B.C., Heiles C. 1991, ApJ, 382, 204 (KH91)

Koo B.C., Kim K.T., Seward F.D. 1995, ApJ, 447, 211 (KKS95)

Koralesky B., Frail D.A., Goss W.M., Claussen M.J., Green A.J. 1998, ApJ, 116, 1323 (K98b)

Kothes R., Fürst E., Reich W. 1998, A&A, 331, 661 (KFR98)

Lacey C.K., Joseph T., Lazio W. Kassim N.E., Duric N., Briggs D.S., Dyer K.K., 2001, ApJ, 559, 954 (L01)

Landecker T.L., Routledge D., Reynolds S.P., Smegal R.J., Borkowski K.J., Seward F.D. 1999, ApJ, 527, 866 (L99)

Leahy D.A. 1986, A&A, 156, 191 (L86)

Leahy D.A. 1989, A&A, 216, 193 (L89)

Leahy D.A., Aschenbach B. 1995, A&A, 293, 853 (LA95)

Leahy D.A., Aschenbach B. 1996, A&A, 315, 260 (LA96)

Levenson N.A., Graham J.R., Snowden S.L. 1999, ApJ, 526, 874 (LGS99)

Lorimer D.R., Lyne A.G., Camilo F. 1998, A&A, 331, 1002 (LLC98)

Lozinskaya T.A. 1981, Soviet Astron. Lett., 7, 17

Matsui Y., Long K.S., Tuohy I.R. 1988, ApJ, 329, 838 (MLT88)

Matthews H.E., Baars J.W.M., Wendker H.J., Goss W.M. 1977, A&A, 55, 1 (M77)

Matthews H.E., Shaver P.A. 1980, A&A, 87, 255 (MS80)

Matthews B.C., Wallace B.J., Taylor A.R. 1998, ApJ, 493, 312 (MWT98)

Mavromatakis F., Boumis P., Paleologou E.V. 2002, A&A, 383, 1011 (MBP02)

Mavromatakis F., Papamastorakis J., Paleologou E.V., Ventura J. 2000, A&A, 353, 371 (M00)

McKee, Christopher F., and Jeremiah P. Ostriker. 1977, AJ, 218, 148

Meaburn J., Allan P.M. 1986, MNRAS, 222, 593 (MA86)

Mills B.Y., Turtle A.J., Little A.G., Durdin J.M. 1984, Australian J. Phys., 37, 321

Milne D.K. 1970, Australian J. Phys., 23, 425

Milne D.K., Roger R.S., Kesteven M.J., Haynes R.F., Wellington K.J., Stewart R.T. 1986, MNRAS, 223, 487 (M86)

Pavlovic M.Z., Urosevic D., Vukotic B., Arbutina B., and Goker U.D., 2013, Astrophys.J.Suppl., 204, 4

Pavlovic M.Z., Dobardzi A., Vukoti B., and Urosevi D., arXiv:1411.2234

Poveda A., Woltjer L. 1968, AJ, 73, 65

Ptuskin V.S., Zirakashvili V.N. 2003, A&A, 403, 1

Rakowski C.E., Hughes J.P., Slane P. 2001, ApJ, 548, 258 (RHS01)

Reich W., Braunsfurth E. 1981, A&A, 99, 17 (RB81)

Reich W., Fürst F., Arnal E.M. 1992, A&A, 256, 214 (RFA92)

Reynoso E.M., Mangum J.G. 2001, ApJ, 121, 347 (RM01)

Rosado M. 1981, ApJ, 250, 222 (R81)

Rosado M. González,J., 1981, Rev. Mexicana. Astron. Astrof., 5, 93 (RG81)

Routledge D., Landecker T.L., Vaneldik J.F. 1986, MNRAS, 221, 809 (RLV86)

Ruiz M.T., May J. 1986, ApJ, 309, 667 (RM86)

Saken J.M., Long K.S., Blair W.P., Winkler P.F. 1995, ApJ, 443, 231 (S95)

Salter C.J., Reynolds S.P., Hogg D.E., Payne J.M., Rhodes P.J. 1989, ApJ, 338, 171 (S89)

Seward F.D., Kearns K.E., Rhode K.L. 1996, ApJ, 471, 887 (SKR96)

Seward F.D., Slane P.O., Smith R.K., Sun M. 2003, ApJ, 584, 414 (S03)

Seward F.D., Velusamy T. 1995, ApJ, 439, 715 (SV95)

Shull J.M., Fesen R.A., Saken J.M. 1989, ApJ, 346, 860 (SFS89)

Slane P., Vancura O., Hughes J.P. 1996, ApJ, 465, 840 (SVH96)

Srinivasan G., Bhattacharya D., Dwarakanath K.S. 1984, J, Astrophys. Astr., 5, 403 (SBD84)

Stil J.M., Irwin J.A. 2001, ApJ, 563, 816 (SI01)

Thorstensen J.R., Fesen R.A., van den Bergh S. 2001, AJ, 122, 297 (TFv01)

Tian W W, Leahy D A, Wang Q D. 2007, A&A, 474, 541 (TL07)

Tian W W, Leahy D A. 2012, MNRAS, 421, 2593 (TL12)

Tian W W, Leahy D A. 2014, ApJL, 783, L2 (TL14)

Uyaniker B., Kothes R., Brunt C.M. 2002, ApJ, 565, 1022 (UKB02)

Völk H.J., Berezhko E.G., Ksenofontov L.T., Rowell G.P. 2002, A&A, 396, 649

Weiler K.W., Sramek R.A. 1988, ARA&A, 26, 295 (WS88)

Xu J. W., Zhang X. Z., Han J. L. 2005, CJAA, 5, 165

Zaninetti L. 2000, A&A, 356, 1023

Zhu H, Tian W W, Zuo P. 2014, ApJ, 793, 95 (ZT14)

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Table 1
Some physical parameters of 57 shell-type Galactic SNRs.

Source	Age	Dist.	Dia.	size	S_{1GHz}	ref
G4.5+6.8	380	2900	3	3	19	H90, G04a
G7.7−3.7	–	4500	29	22	11	M86
G8.7−0.1	15800	3900	51	45	80	G96
G18.8+0.3	16000	12000	57	17x11	33	G04a, TL07
G27.4+0.0	2700	6800	8	4	6	G04a, C82
G31.9+0.0	4500	7200	13	7x5	25	CS01, G14
G32.8−0.1	–	7100	35	17	11	K98b
G33.6+0.1	9000	7800	23	10	20	S03, SV95, G04a, G14
G39.2−0.3	1000	11000	22	8x6	18	G14, C82
G41.1−0.3	1400	8000	8	4.5x2.5	25	C99, C82, B82, G14
G43.3−0.2	3000	10000	10	4x3	38	L01, ZT14
G49.2−0.7	30000	6000	52	30	160	KKS95, G04a
G53.6−2.2	15000	2800	24	33x28	8	S95, G04a
G55.0+0.3	1100000	14000	71	20x15	0.5	MWT98
G65.3+5.7	14000	1000	78	310x240	42	LRH80, G14, R81
G73.9+0.9	10000	1300	8	27	9	LLC98, G14, L89
G74.0−8.5	14000	400	23	230x160	210	LGS99, SI01, G04a
G78.2+2.1	50000	1500	26	60	320	LLC98, KH91, G14
G84.2−0.8	11000	4500	23	20x16	11	MS80, M77, G04a
G89.0+4.7	19000	800	24	120x90	220	LA96
G93.3+6.9	5000	2200	15	27x20	9	L99, G04a
G93.7−0.2	–	1500	35	80	65	UKB02
G109.1−1.0	17000	4000	24	28	22	FH95, G04a, G14, HHv81, TL12
G111.7−2.1	320	3400	5	5	2720	TFv01
G114.3+0.3	41000	700	15	90x55	5.5	MBP02, G04a, G14
G116.5+1.1	280000	1600	32	80x60	10	G04a, G14, RB81
G116.9+0.2	44000	1600	16	34	8	KH91, G04a, G14
G119.5+10.2	24500	1400	37	90	36	M00
G120.1+1.4	410	2300	5	8	56	H90, G04a
G127.1+0.5	85000	5250	69	45	12	G14, FRS84
G132.7+1.3	21000	2200	51	80	45	G04a, GTG80
G156.2+5.7	26000	2000	64	110	5	RFA92
G160.9+2.6	7700	1000	38	140x120	110	LA95
G166.0+4.3	81000	4500	57	55x35	7	KH91, G04a, L89
G166.2+2.5	150000	8000	186	90x70	11	G14, RLV86
G182.4+4.3	3800	3000	44	50	0.4	KFR98, G14
G205.5+0.5	50000	1600	102	220	140	CB99, G14
G206.9+2.3	60000	7000	102	60x40	6	G14, L86
G260.4−3.4	3400	2200	35	60x50	130	B94, RG81
G266.2−1.2	680	1500	52	120	50	K02, AIS99
G272.2−3.2	6000	1800	8	15	0.4	D97

Table 1—*Continued*

Source	Age	Dist.	Dia.	size	S_{1GHz}	ref
G284.3−1.8	10000	2900	20	24	11	G14, RM86
G296.5+10.0	20000	2000	44	90x65	48	G14, MLT88
G296.8−0.3	1600000	9600	47	20x14	9	GJ95, G04a
G299.2−2.9	5000	500	2	18x11	0.5	SVH96
G309.2−0.6	2500	4000	16	15x12	7	RHS01
G315.4−2.3	2000	2300	28	42	49	DSM01, G04a
G321.9−0.3	200000	9000	70	31x23	13	G14, SFS89, S89
G327.4+0.4	–	4800	29	21	30	SKR96, G04a, G14, WS88
G327.6+14.6	980	2200	19	30	19	G04a, SBD84
G330.0+15.0	–	1200	63	180	350	K96
G332.4−0.4	2000	3100	9	10	28	CDB97, G04a, MA86
G337.2−0.7	3250	15000	26	6	1.5	RHS01, G14
G337.8−0.1	–	12300	27	9x6	18	K98b
G346.6−0.2	–	8200	19	8	8	K98b, D93
G349.7+0.2	14000	11500	9	2.5x2	20	RM01, G04a, TL14
G352.7−0.1	2200	8500	17	8x6	4	K98a

[a]Many of the radio SNRs have more than one published value for distance and age. For these, we either chose the most recent estimates or used an average of the available estimates, or the most commonly adopted value.

[b]Diameters were calculated using from distances together with the angular sizes in Green (2004, 2009 & 2014) catalogue. In addition, some data have been updated according to the new results in Green (2014) and other new references.

[c]Some data regarding G349.7+0.2 (TL14), G43.3-0.2 (ZT14), G18.8+0.3 (TL07) and G109.1-1.0 (TL12) have been updated.